iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Specific heat in the $SU(N)$ Heisenberg spin-glass model

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2004 J. Phys.: Condens. Matter 16 S723

(http://iopscience.iop.org/0953-8984/16/11/021)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 27/05/2010 at 12:53

Please note that terms and conditions apply.

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) S723–S727 PII: S0953-8984(04)74124-7

Specific heat in the $SU(N)$ Heisenberg spin-glass model

M J Rozenberg and A Camjayi

Departamento de Física, FCEN, Universidad de Buenos Aires, Ciudad Universitaria Pab I, Buenos Aires, 1428, Argentina

E-mail: marcelo@df.uba.ar

Received 7 January 2004
Published 4 March 2004
Online at stacks.iop.org/JPhysCM/16/S723 (DOI: 10.1088/0953-8984/16/11/021)

Abstract
We solve the $SU(N)$ Heisenberg spin-glass model in the limit of large N focusing on the small temperature behaviour of the specific heat C_V within the spin-glass phase. We consider the influence of the quantum fluctuations and observe that when they are strong the low T behaviour is quadratic. As quantum fluctuations decrease for large values of S, a broad maximum appears in the $C_V(T)/T$ curve.

1. Introduction

The understanding of the competition between disorder, quantum and thermal fluctuations remains among the challenging problems of condensed matter physics [1–4]. These three aspects are always present to some extent in experiments on real systems; therefore, a clear understanding of their interplay is very desirable. In systems where disorder is relevant we usually encounter the phenomenology of slow dynamics that is associated with glassy states. When quantum fluctuations become important, phases with glassy orders can be driven to more conventional phases through interesting quantum phase transitions [5]. Perhaps the archetype of frustrated quantum magnets is the bi-layer Kagomé lattice $SrCr_{9p}Ga_{12-9p}O_{19}$ (SCGO) that only becomes a spin-glass at the low temperature of about 5 K [6–10]. In sharp contrast to ordinary classical spin-glasses, SCGO exhibits some unusual remarkable features that are associated with strong quantum fluctuations: the magnetic fluctuation spectrum, $\chi''(\omega)$, is found to vanish linearly in ω at low frequencies [8], and the specific heat is proportional to T^2 [7]. On the theoretical side, these observations have remained largely unaccounted for.

Progress in the understanding of models of disordered quantum magnets in finite dimensions is rather slow. In fact, a great deal of our knowledge still relies on solutions of systems with long-ranged interactions. These mean-field models are appealing because they are mathematically more tractable while retaining much of the physics associated with slow dynamics. Among the simplest mean field models for quantum spin-glasses, the quantum version of the Sherrington–Kirkpatrick (SK) model has received a great deal of attention. It

© 2004 IOP Publishing Ltd Printed in the UK

is a Heisenberg model with Gaussianly distributed random interactions between all pairs of spins in the lattice. The model was first considered by Bray and Moore [11] and they predicted a spin-glass phase at low temperature, with a freezing temperature T_g substantially reduced from the usual (Ising) version of the SK model. Later, Sachdev and Ye [12] introduced a generalization of the model to $SU(N)$ spins which could be studied in the large N limit. They found a very interesting spin-liquid phase down to $T = 0$. In more recent work on this model, a generalized phase diagram as a function of T and S was obtained using a bosonic representation [13–15].

The spin quantum number S can be thought of as a parameter that controls the strength of the quantum fluctuations. For $S \to \infty$ one goes to the 'classical' limit while for small S the quantum fluctuations are strongest. A low temperature spin-glass phase was found for all non-zero S and $T_g \sim S^2$ at large S [13–15]. Remarkably, the spin-liquid phase was also found at very low spin S [12–14]. Therefore, quantum fluctuations can drive the model through an interesting quantum critical point between a spin-liquid state at $S \to 0$ and a quantum spin-glass for finite S. Recent numerical studies based on quantum Monte Carlo and exact diagonalization techniques for the $SU(2)$ model have validated some aspects of previous investigations [16–18].

The goal of the present work is to focus on the behaviour of the specific heat $C_V(T)$ as a function of the strength of the quantum fluctuations. This can be done in detail for the solvable $SU(N)$ model in the limit of large N [13, 14, 19]. We find that $C_V(T)/T$ is *linear* at small T when S is small and develops a broad maximum *beneath* T_g for larger values of S. Interestingly, the linear with T behaviour for $C_V(T)/T$ has been a puzzling experimental finding in SCGO [7]. We shall also present an intuitive argument to understand our findings.

2. The $SU(N)$ Heisenberg spin-glass model

The model Hamiltonian is

$$H = \frac{1}{\sqrt{\mathcal{N}N}} \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j, \tag{1}$$

where the magnetic exchange couplings J_{ij} are independent, quenched random variables distributed according to a Gaussian distribution where J is the standard deviation and the unit of energy. As already pointed out by Bray and Moore [11], one uses the replica trick to average over the disorder [2] and the lattice infinite-range model maps exactly onto a *self-consistent single site model* with the action (in imaginary time τ, with β the inverse temperature)

$$S_{\text{eff}} = S_{\text{B}} - \frac{J^2}{2N} \int_0^\beta \mathrm{d}\tau\, \mathrm{d}\tau'\, Q^{ab}(\tau - \tau') \vec{S}^a(\tau) \cdot \vec{S}^b(\tau') \tag{2}$$

and the self-consistency condition

$$Q^{ab}(\tau - \tau') = \frac{1}{N^2} \langle \vec{S}^a(\tau) \cdot \vec{S}^b(\tau') \rangle_{S_{\text{eff}}} \tag{3}$$

where $a, b = 1, \ldots, n$ denote the replica indices (the limit $n \to 0$ has to be taken later) and S_{B} is the Berry phase of the spin [12]. Due to the time dependence, the solution of these mean-field equations remains a very difficult problem for $N = 2$, even in the paramagnetic phase [16].

We shall use the bosonic representation [12–15] for the spin operators, where S is represented with Schwinger bosons b by $S_{\alpha\beta} = b_\alpha^\dagger b_\beta - S\delta_{\alpha\beta}$, with the constraint $\sum_\alpha b_\alpha^\dagger b_\alpha = SN$ $(0 \leqslant S)$. In the language of Young tableaux, these representations are described by one line of length SN. They are a natural generalization of an $SU(2)$ spin of size S.

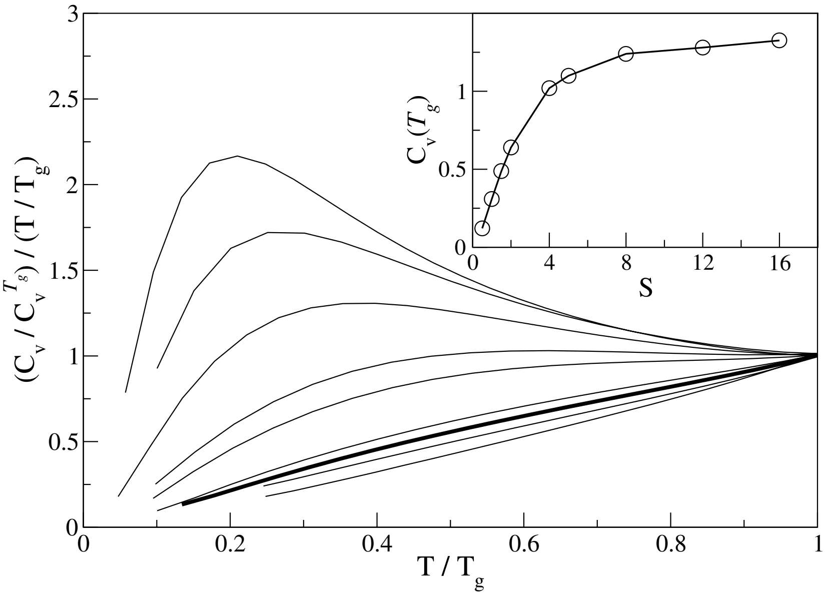

Figure 1. The specific heat divided by the temperature C_V/T as a function of the temperature for various values of the spin S in the spin-glass phase. The x-axis has been normalized by $T_g(S) \sim S^2$ and the y-axis by the specific heat at T_g and the corresponding value of the spin $C_V^{T_g}(S)$ for clearer visualization. The results corresponds to $S = 0.5$, 1, 1.5, 2, 4, 5, 8, 12 and 16 (bottom to top). The results for $S = 3/2$ that are relevant for the SCGO system are highlighted by the thick curve. The inset shows the behaviour of the specific heat at T_g, $C_V^{T_g}$, as a function of S.

In the $N \to \infty$ limit, the mean field self-consistent model (2)–(3) reduces to an integral equation for the Green function of the boson $G_b^{ab}(\tau) \equiv -\overline{\langle T b^a(\tau) b^{\dagger b}(0)\rangle}$ where the bar denotes the average over disorder and the brackets the thermal average [12]:

$$(G_b^{-1})^{ab}(\mathrm{i}\nu_n) = \mathrm{i}\nu_n \delta_{ab} + \lambda^a \delta_{ab} - \Sigma_b^{ab}(\mathrm{i}\nu_n) \tag{4}$$

$$\Sigma_b^{ab}(\tau) = J^2 (G_b^{ab}(\tau))^2 G_b^{ab}(-\tau) \tag{5}$$

$$G_b^{aa}(\tau = 0^-) = -S. \tag{6}$$

The *local* spin susceptibility $\chi_{\mathrm{loc}}(\tau) = \langle S(\tau) S(0)\rangle$ is given in the large N limit by $\chi_{\mathrm{loc}}(\tau) = G_b^{aa}(\tau) G_b^{aa}(-\tau)$.

In the spin-glass phase it is enough to perform a one step symmetry broken solution [13–15]. Equations (4)–(6) were solved self-consistently on the Matsubara axis. The general form of the spin susceptibility can written as $\chi''_{\mathrm{loc}}(\omega) = q_{EA}\delta(\omega) + \chi''_{\mathrm{reg}}(\omega)$, where q_{EA} is the spin glass order parameter [13, 14, 19].

3. The specific heat

The specific heat can be computed by taking the numerical derivative of the energy of the system [13],

$$E(T) = -\frac{J^2}{2} \int_0^\beta \mathrm{d}\tau [G^{ab}(\tau) G^{ab}(-\tau)]^2. \tag{7}$$

The results for the spin-glass phase are shown in figure 1 for various values of S. We have scaled the x-axis by $T_g \propto S^2$ and the y-axis by $C_V(T_g)$ for clearer visualization. When S is small, C_V/T is linear in T up to T_g. However, when S increases, reducing the importance of the quantum fluctuations, there is a dramatic change of behaviour, with a broad maximum appearing in the C_V/T curves. The position of this maximum shifts down to 0 for increasing S.

An interesting aspect of this result is that a linear form of C_V/T has been found in the SCGO compound, and has remained as a puzzling result [7]. In SCGO the active magnetic sites are Cr^{3+} with $S = 3/2$. Our model results for $S = 3/2$ are highlighted in bold in figure 1. The results show that the linear regime of C_V/T extends all the way up to T_g. This observation is also true for the experimental behaviour reported in SCGO [7], so our model results suggests that SCGO should be considered as a rather *small S* system with strong quantum fluctuations.

In what follows we shall argue that this behaviour can be understood in terms of other rather intuitive results of the model, namely the behaviour of the order parameter q_{EA} and the dynamical spin susceptibility $\chi''(\omega)$.

In a previous work [13, 19] it was found that the order parameter at low T follows a simple quadratic behaviour $q_{EA}(T) = q_{EA}(0)(1 - \alpha T^2)$, with α an S-dependent constant. On the other hand, it was also found [19] that $\chi''(\omega)$ is of the form $q_{EA}\delta(\omega) + \chi''_{\rm reg}(\omega)$, where the regular part shows a pseudogap in the limit of $T \to 0$ with a low frequency behaviour given by $\propto\omega/S$. As the temperature is increased, within the spin-glass ordered phase, the δ-function contribution due to the frozen moments gradually melts and its spectral weight partially fills up the pseudogap. The width of the associated excitations is given by T [19] as one would expect for diffusive modes [20]. One can now combine these results with the expression for the energy [16, 20]

$$U = \int \omega\chi''(\omega)\,\mathrm{d}\omega = \int \omega(1 - \mathrm{e}^{-\omega/T})\rho(\omega)\,\mathrm{d}\omega \qquad (8)$$

where $\rho(\omega)$ is the spectral density whose integral obeys the sum-rule $S(S+1)$. We can thus argue that as T increases from 0, an amount proportional to T^2 of the spectral weight of the $\delta(\omega)$ will melt and produce a regular contribution to the low frequency part of $\rho(\omega)$ with width $\sim T$ and spectral weight $\sim T^2$. Expanding the exponential in (8), we find that the change in energy is approximately given by

$$\Delta U = \int_0^T \omega\left(\frac{\omega}{T}\right)T\,\mathrm{d}\omega \sim T^3. \qquad (9)$$

Since $C_V(T) = \partial U/\partial T$ the quadratic form for $C_V(T)$ follows.

4. Conclusion

We have obtained the detailed behaviour of the low temperature specific heat within the spin-glass phase of a disordered quantum magnet model that can be solved analytically.

We found that when the quantum fluctuations are strongest for small values of S, $C_V(T)/T$ shows a simple linear with T behaviour up to T_g. When quantum fluctuations are reduced, a broad maximum in $C_V(T)/T$ appears and moves down in T.

Interestingly, the linear with T behaviour up to T_g has been reported [7] in the $SrCr_{9p}Ga_{12-9p}O_{19}$ compound for p ranging from 0.3 to 0.98, and has remained largely unaccounted for. Our model results seem to indicate that SCGO should be considered as a small S system with strong quantum fluctuations.

References

[1] Fischer K H and Hertz J A 1991 *Spin Glasses* (Cambridge: Cambridge University Press)
[2] Mézard M, Parisi G and Virasoro M 1987 *Spin Glass Theory and Beyond* (Singapore: World Scientific)

[3] Rieger H and Young A P 1997 Quantum spin glasses *Complex Behaviour of Glassy Systems* (*Springer Lecture Notes in Physics* vol 492) (Berlin: Springer) p 254
[4] Binder K and Young A P 1986 *Rev. Mod. Phys.* **58** 801
[5] Sachdev S 1999 *Quantum Phase Transitions* (Cambridge: Cambridge University Press)
[6] Obradors X *et al* 1990 *Solid State Commun.* **65** 189
[7] Ramirez A *et al* 1992 *Phys. Rev.* B **45** 2505
Ramirez A *et al* 2000 *Phys. Rev. Lett.* **84** 2957
[8] Lee S-H *et al* 1996 *Europhys. Lett.* **35** 127
[9] Mondelli C *et al* 1999 *Physica* B **266** 104
[10] Limot L *et al* 2002 *Phys. Rev.* B **65** 144447 and references therein
[11] Bray A J and Moore M A 1980 *J. Phys. C: Solid State Phys.* **13** L655
[12] Sachdev S and Ye J 1993 *Phys. Rev. Lett.* **70** 3339
[13] Georges A, Parcollet O and Sachdev S 2000 *Phys. Rev. Lett.* **85** 840
[14] Georges A, Parcollet O and Sachdev S 2001 *Phys. Rev.* B **63** 134406
[15] Kopeć T K 1995 *Phys. Rev.* B **52** 9590
[16] Grempel D R and Rozenberg M J 1998 *Phys. Rev. Lett.* **80** 389
[17] Arrachea L and Rozenberg M J 2001 *Phys. Rev. Lett.* **86** 5172
[18] Arrachea L and Rozenberg M J 2002 *Phys. Rev.* B **65** 224430
[19] Camjayi A and Rozenberg M 2003 *Phys. Rev. Lett.* **90** 217202
[20] Forster D 1990 *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions* (Reading, MA: Addison-Wesley)